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Crank-Nicolson Scheme for Numerical Solutions of Two-dimensional Coupled Burgers' Equations

Vineet Kumar Srivastava, Mohammad Tamsir, Utkarsh Bhardwaj, YVSS Sanyasiraju

Abstract— The two-dimensional Burgers' equation is a mathematical model to describe various kinds of phenomena such as turbulence and viscous fluid. In this paper, Crank-Nicolson finite-difference method is used to handle such problem. The proposed scheme forms a system of nonlinear algebraic difference equations to be solved at each time step. To linearize the non-linear system of equations, Newton's method is used. The obtained linear system is then solved by Gauss elimination with partial pivoting. The proposed scheme is unconditionally stable and second order accurate in both space and time. Numerical results are compared with those of exact solutions and other available results for different values of Reynolds number. The proposed method can be easily implemented for solving nonlinear problems evolving in several branches of engineering and science.

Keywords — Burgers' equations; Crank-Nicolson scheme; finite- difference; Newton's method; Reynolds number

1 INTRODUCTION

THE nonlinear coupled Burgers' equation is a special form of incompressible Navier-Stokes equation without having pressure term and continuity equation. Burgers' equation is an important partial differential equation from fluid dynamics, and is widely used for various physical applications, such as modeling of gas dynamics and traffic flow, shock waves [1], investigating the shallow water waves[2,3], in examining the chemical reaction-diffusion model of Brusselator[4] etc. It is also used to test several numerical algorithms. The first attempt to solve Burgers' equation analytically was given by Bateman [5], who derived the steady solution for a simple one-dimensional Burgers' equation, which was used by J.M. Burger in [6] to model turbulence. In the past several years, numerical solution to one-dimensional Burgers' equation and system of multidimensional Burgers' equations have attracted a lot of attention and which has resulted in various finite-difference, finite-element and boundary element methods. Finite difference methods can be classified in two broad categories, i.e. explicit and implicit. Chabak and Sharma [7] have executed the solution of two dimensional coupled wave eqution explicitly. An implicit approach has been utilized for solving two dimensional coupled Burgers' equations. Since in this paper the focus is numerical solutions of the two-dimensional Burgers' equations, a detailed survey of the numerical schemes for solving the one-dimensional Burgers' equation is not necessary. Interested readers can refer to [8-14] for more details.

Consider two-dimensional coupled nonlinear Burgers' equations

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=0, \quad (1)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}-\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)=0, \quad (2)$$

subject to the initial conditions

$$u(x,y,0)=\psi_1(x,y);\ (x,y)\in\Omega,$$
$$v(x,y,0)=\psi_2(x,y);\ (x,y)\in\Omega,$$

and boundary conditions

$$u(x,y,t)=\xi(x,y,t);\ (x,y)\in\partial\Omega,\ t>0,$$
$$v(x,y,t)=\zeta(x,y,t);\ (x,y)\in\partial\Omega,\ t>0,$$

Where $\Omega=\{(x,y): a\le x\le b,\ c\le y\le d\}$ and $\partial\Omega$ is its boundary; $u(x,y,t)$ and $v(x,y,t)$ are the velocity components to be determined, ψ_1,ψ_2, ξ and ζ are known functions and Re is the Reynolds number.

The analytic solution of eqns. (1) and (2) was proposed by Fletcher using the Hopf-Cole transformation [15]. The numerical solutions of this system of equations have been

- Vineet Kumar Srivastava, Faculty member, Department of Mathematics, The ICFAI University, Dehradun, India, Email: vineetsriiitm@gmail.com
- Mohammad Tamsir, Faculty member, Department of Mathematics, The ICFAI University, Dehradun, India, Email: tamsiriitm@gmail.com.
- Utkarsh Bhardwaj, third year B.Tech. Computer Science student, Department of Computer Science, The ICFAI University, Dehradun, India, Email: hap_tork@hotmail.com.
- YVSS Sanyasiraju, Professor, Department of Mathematics, IIT Madras Chennai, India. E-mail: sryedida@iitm.ac.in.

solved by many researchers. Jain and Holla [16] developed two algorithms based on cubic spline method. Fletcher [17] has discussed the comparison of a number of different numerical approaches.Wubs and Goede [18] have applied an explicit–implicit method. Goyon [19] used several multilevel schemes with ADI. Recently A. R. Bahadır [20] has applied a fully implicit method.
In this paper, Crank-Nicolson finite-difference scheme is used for solving two-dimensional coupled nonlinear Burgers' equations. Two numerical examples have been carried out and their results are presented to illustrate the efficiency of the proposed method.

2 The Solution procedure

The computational domain Ω is discretized with uniform grid. Denote the discrete approximation of $u(x,y,t)$ and $v(x,y,t)$ at the grid point $(i\Delta x, j\Delta y, n\Delta t)$ by $u_{i,j}^{n}$ and $v_{i,j}^{n}$ respectively $(i=0,1,2......,n_x;\ j=0,1,2.....,n_y;$ $n=0,1,2......),$ where $\Delta x = 1/n_x$ is the grid size in x-direction, $\Delta y = 1/n_y$ is the grid size in y-direction, and Δt represents the increment in time.
Crank-Nicolson finite-difference approximation to (1) is given by

$$\frac{u_{i,j}^{n+1}-u_{i,j}^{n}}{\Delta t}+\frac{1}{2}[\ u_{i,j}^{n+1}(\frac{u_{i+1,j}^{n+1}-u_{i-1,j}^{n+1}}{2\Delta x})+u_{i,j}^{n}(\frac{u_{i+1,j}^{n}-u_{i-1,j}^{n}}{2\Delta x})\ ]$$

$$+\frac{1}{2}[\ v_{i,j}^{n+1}(\frac{u_{i,j+1}^{n+1}-u_{i,j-1}^{n+1}}{2\Delta y})+v_{i,j}^{n}(\frac{u_{i,j+1}^{n}-u_{i,j-1}^{n}}{2\Delta y})\ ]$$
$$-\frac{1}{\mathrm{Re}}[\frac{1}{2}\{(\frac{u_{i+1,j}^{n+1}-2u_{i,j}^{n+1}+u_{i-1,j}^{n+1}}{(\Delta x)^2})+(\frac{u_{i+1,j}^{n}-2u_{i,j}^{n}+u_{i-1,j}^{n}}{(\Delta x)^2})\}$$

$$+\frac{1}{2}\{(\frac{u_{i,j+1}^{n+1}-2u_{i,j}^{n+1}+u_{i,j-1}^{n+1}}{(\Delta y)^2})+(\frac{u_{i,j+1}^{n}-2u_{i,j}^{n}+u_{i,j-1}^{n}}{(\Delta y)^2})\}]=0$$

Similarly, Crank-Nicolson approximation to (2) is given by

$$\frac{v_{i,j}^{n+1}-v_{i,j}^{n}}{\Delta t}+\frac{1}{2}[\ u_{i,j}^{n+1}(\frac{v_{i+1,j}^{n+1}-v_{i-1,j}^{n+1}}{2\Delta x})+u_{i,j}^{n}(\frac{v_{i+1,j}^{n}-v_{i-1,j}^{n}}{2\Delta x})\ ]$$

$$+\frac{1}{2}[\ v_{i,j}^{n+1}(\frac{v_{i,j+1}^{n+1}-v_{i,j-1}^{n+1}}{2\Delta y})+v_{i,j}^{n}(\frac{v_{i,j+1}^{n}-v_{i,j-1}^{n}}{2\Delta y})\ ]$$
$$-\frac{1}{\mathrm{Re}}[\frac{1}{2}\{(\frac{v_{i+1,j}^{n+1}-2v_{i,j}^{n+1}+v_{i-1,j}^{n+1}}{(\Delta x)^2})+(\frac{v_{i+1,j}^{n}-2v_{i,j}^{n}+v_{i-1,j}^{n}}{(\Delta x)^2})\}$$
$$+\frac{1}{2}\{(\frac{v_{i,j+1}^{n+1}-2v_{i,j}^{n+1}+v_{i,j-1}^{n+1}}{(\Delta y)^2})+(\frac{v_{i,j+1}^{n}-2v_{i,j}^{n}+v_{i,j-1}^{n}}{(\Delta y)^2})\}]=0$$

Consider the above nonlinear system of equations in the form

$$\Theta\ (\ w\ )\ =\ 0\ , \qquad (3)$$

Where, $\Theta = (g_1, g_2, ..., g_{2n})^t$, $w = (u_1^{n+1}, v_1^{n+1}, u_2^{n+1}, v_2^{n+1}, ..., u_n^{n+1}, v_n^{n+1}),$
and $n = n_x - 1 = n_y - 1.$ Newton's method when it is applied to (3) results in the following steps:

1. set $w^{(0)}$, an initial approximation.
2. while for $k = 0,1,.......$ until convergence do:
   - solve $J(w^{(k)})\Delta w^{(k)} = -\Theta(w^{(k)}),$
   - set $w^{(k+1)} = w^{(k)} + \Delta w^{(k)},$

Where $J(w^{(k)})$ is the Jacobian matrix and $\Delta w^{(k)}$ is the correction vector. It is a $n \times n$ square matrix whose elements are blocks of size 2×2. Newton's iteration at each time-step is stopped when $\|\Theta(w^{(k)})\|_\infty \le 10^{-5}$.
By using Taylor's series expansion one can easily see that the present scheme has accuracy of order $\mathrm{O}((\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2)$.

3 Numerical examples and discussion

3.1 Problem 1

The exact solutions of Burgers' equations (1) and (2) can be generated by using the Hopf–Cole transformation [13] which are:

$$u(x,y,t) = \frac{3}{4} - \frac{1}{4[1+\exp(\ (-4x+4y-t)\,\mathrm{Re}/32\ )]},$$
$$v(x,y,t) = \frac{3}{4} + \frac{1}{4[1+\exp(\ (-4x+4y-t)\,\mathrm{Re}/32\ )]},$$

Here the computational domain is taken as a square domain $\Omega = \{(x,y): 0 \le x \le 1, 0 \le y \le 1\}$. The initial and boundary conditions for $u(x,y,t)$ and $v(x,y,t)$ are taken from the analytical solutions. The numerical computations are performed using uniform grid, with a mesh width $\Delta x = \Delta y = 0.05$. From Tables 1-10, it is clear that the results from the present study are in good agreement with the exact solution for different values of Reynolds number. From Tables 1-10, it is also clear that that the present scheme is unconditionally stable as it is accurate for any time step-size. Perspective views of u and v for Re=500 at $t = 0.5$ with $\Delta t = 0.001$ are given in Figs. 1 and 2.

3.2 Problem 2.

Here the computational domain is taken as $\Omega = \{(x, y) : 0 \le x \le 0.5, 0 \le y \le 0.5\}$ and Burgers' equations (1) and (2) are taken with the initial conditions,

$$\left.\begin{aligned} u(x,y,0) &= \sin(\pi x) + \cos(\pi y) \\ v(x,y,0) &= x + y \end{aligned}\right\} 0 \le x \le 0.5,\ 0 \le y \le 0.5,$$

and boundary conditions,

$$\left.\begin{aligned} u(0,y,t) &= \cos(\pi y), \quad u(0.5,y,t) = 1 + \cos(\pi y) \\ v(0,y,t) &= y, \qquad\quad v(0.5,y,t) = 0.5 + y \end{aligned}\right\} 0 \le y \le 0.5,\ t \ge 0,$$

$$\left.\begin{aligned} u(x,0,t) &= 1 + \sin(\pi x), \quad u(x,0.5,t) = \sin(\pi x) \\ v(x,0,t) &= x, \qquad\qquad v(x,0.5,t) = x + 0.5 \end{aligned}\right\} 0 \le x \le 0.5,\ t \ge 0,$$

The numerical computations are performed using 20×20 grids and $\Delta t = 0.0001$. The steady state solutions for $\text{Re} = 50$ and $\text{Re} = 500$ are obtained at $t = 0.625$. Perspective views of u and v for $\text{Re} = 50$ at $\Delta t = 0.0001$ are given in Fig.s 3 and 4. The results given in Tables 11- 14 demonstrate that the proposed scheme achieves similar results given by [16,20].

Table 1
The numerical results for u in comparison with the exact solution at $t = 0.5$ and $t = 2$ with $\Delta t = 0.0001$, and $\text{Re} = 10$.

(x, y)	t=0.5		t=2	
	Numerical	**Exact**	**Numerical**	**Exact**
(0.1, 0.1)	0.61525	0.61525	0.58716	0.58716
(0.5, 0.1)	0.58540	0.58540	0.56129	0.56127
(0.9, 0.1)	0.56062	0.55984	0.54330	0.54113
(0.3, 0.3)	0.61526	0.61525	0.58718	0.58716
(0.7, 0.3)	0.58555	0.58540	0.56188	0.56127
(0.1, 0.5)	0.64628	0.64628	0.61721	0.61720
(0.5, 0.5)	0.61529	0.61525	0.58738	0.58716
(0.9, 0.5)	0.58707	0.58540	0.56654	0.56127
(0.3, 0.7)	0.64638	0.64628	0.61771	0.61720
(0.7, 0.7)	0.61562	0.61525	0.58900	0.58716
(0.1, 0.9)	0.67557	0.67481	0.65097	0.64817
(0.5, 0.9)	0.64773	0.64628	0.62264	0.61720
(0.9, 0. 9)	0.61802	0.61525	0.59642	0.58716

Table 2
The numerical results for v in comparion with the exact solution at $t = 0.5$ and $t = 2$ with $\Delta t = 0.0001$, and $\text{Re} = 10$.

(x, y)	t=0.5		t=2	
	Numerical	**Exact**	**Numerical**	**Exact**
(0.1, 0.1)	0.88475	0.88475	0.91284	0.91284
(0.5, 0.1)	0.91460	0.91460	0.93871	0.93873
(0.9, 0.1)	0.93938	0.94016	0.95670	0.95887
(0.3, 0.3)	0.88474	0.88475	0.91283	0.91284
(0.7, 0.3)	0.91445	0.91460	0.93812	0.93873
(0.1, 0.5)	0.85372	0.85373	0.88279	0.88280
(0.5, 0.5)	0.88471	0.88475	0.91262	0.91284
(0.9, 0.5)	0.91293	0.91460	0.93346	0.93873
(0.3, 0.7)	0.85362	0.85373	0.88229	0.88280
(0.7, 0.7)	0.88437	0.88475	0.91101	0.91284
(0.1, 0.9)	0.82443	0.82519	0.84903	0.85183
(0.5, 0.9)	0.85227	0.85373	0.87736	0.88280
(0.9, 0.9)	0.88198	0.88475	0.90358	0.91284

Table 3
The numerical results for u in comparison with the exact solution at $t = 0.5$ and $t = 2$ with $\Delta t = 0.0001$, and $\text{Re} = 100$.

(x, y)	t=0.5		t=2	
	Numerical	**Exact**	**Numerical**	**Exact**
(0.1, 0.1)	0.54300	0.54332	0.500470	0.50048
(0.5, 0.1)	0.50034	0.50035	0.500003	0.50000
(0.9, 0.1)	0.50000	0.50000	0.500000	0.50000
(0.3, 0.3)	0.54269	0.54332	0.500441	0.50048
(0.7, 0.3)	0.50032	0.50035	0.500003	0.50000
(0.1, 0.5)	0.74215	0.74221	0.555151	0.55568
(0.5, 0.5)	0.54250	0.54332	0.500415	0.50048
(0.9, 0.5)	0.50030	0.50035	0.500014	0.50000
(0.3, 0.7)	0.74212	0.74221	0.554811	0.55568
(0.7, 0.7)	0.54246	0.54332	0.500683	0.50048
(0.1, 0.9)	0.74995	0.74995	0.744215	0.74426
(0.5, 0.9)	0.74213	0.74221	0.559802	0.55568
(0.9, 0.9)	0.54640	0.54332	0.513409	0.50048

Table 4
The numerical results for v in comparison with the exact solution at $t = 0.5$ and $t = 2$ with $\Delta t = 0.0001$, and $\text{Re} = 100$.

(x, y)	t=0.5		t=2	
	Numerical	**Exact**	**Numerical**	**Exact**
(0.1, 0.1)	0.95700	0.95668	0.99953	0.99952
(0.5, 0.1)	0.99966	0.99965	1.00000	1.00000
(0.9, 0.1)	1.00000	1.00000	1.00000	1.00000
(0.3, 0.3)	0.95731	0.95668	0.99956	0.99952
(0.7, 0.3)	0.99968	0.99965	1.00000	1.00000
(0.1, 0.5)	0.75785	0.75779	0.94485	0.94433
(0.5, 0.5)	0.95750	0.95668	0.99959	0.99952
(0.9, 0.5)	0.99970	0.99965	0.99999	1.00000
(0.3, 0.7)	0.75789	0.75779	0.94519	0.94433
(0.7, 0.7)	0.95754	0.95668	0.99932	0.99952
(0.1, 0.9)	0.75006	0.75005	0.75579	0.75574
(0.5, 0.9)	0.75787	0.75779	0.94020	0.94433
(0.9, 0.9)	0.95360	0.95668	0.98659	0.99952

Table 5
The numerical results for u in comparison with the exact solution at $t = 0.5$ and $t = 2$ with $\Delta t = 0.0001$, and $\mathrm{Re} = 500$.

(x, y)	t=0.5		t=2	
	Numerical	**Exact**	**Numerical**	**Exact**
(0.1, 0.1)	0.48732	0.50010	0.49725	0.50000
(0.5, 0.1)	0.50002	0.50000	0.50024	0.50000
(0.9, 0.1)	0.50000	0.50000	0.49932	0.50000
(0.3, 0.3)	0.49531	0.50010	0.50687	0.50000
(0.7, 0.3)	0.50001	0.50000	0.49929	0.50000
(0.1, 0.5)	0.74990	0.75000	0.43945	0.50048
(0.5, 0.5)	0.49438	0.50010	0.49958	0.50000
(0.9, 0.5)	0.49977	0.50000	0.51378	0.50000
(0.3, 0.7)	0.75001	0.75000	0.41654	0.50048
(0.7, 0.7)	0.49323	0.50010	0.51054	0.50000
(0.1, 0.9)	0.75000	0.75000	0.75003	0.75000
(0.5, 0.9)	0.75001	0.75000	0.42895	0.50048
(0.9, 0.9)	0.47390	0.50010	0.56293	0.50000

Table 6
The numerical results for v in comparison with the exact solution at $t = 0.5$ and $t = 2$ with $\Delta t = 0.0001$, and $\mathrm{Re} = 500$.

(x, y)	t=0.5		t=2	
	Numerical	**Exact**	**Numerical**	**Exact**
(0.1, 0.1)	1.01268	0.99990	1.00276	1.00000
(0.5, 0.1)	0.99999	1.00000	0.99975	1.00000
(0.9, 0.1)	1.00000	1.00000	1.00068	1.00000
(0.3, 0.3)	1.00469	0.99990	0.99313	1.00000
(0.7, 0.3)	0.99999	1.00000	1.00072	1.00000
(0.1, 0.5)	0.75010	0.75000	1.06055	0.99952
(0.5, 0.5)	1.00562	0.99990	1.00041	1.00000
(0.9, 0.5)	1.00023	1.00000	0.98622	1.00000
(0.3, 0.7)	0.74999	0.75000	1.08346	0.99952
(0.7, 0.7)	1.00677	0.99990	0.98946	1.00000
(0.1, 0.9)	0.75000	0.75000	0.74997	0.75000
(0.5, 0.9)	0.74999	0.75000	1.07105	0.99952
(0.9, 0.9)	1.02610	0.99990	0.93707	1.00000

Table 7
The numerical values for u in comparison with the exact solution at $t = 0.5$ and $t = 2$ with $\Delta t = 0.001$, and $\mathrm{Re} = 500$.

(x, y)	t=0.5		t=2	
	Numerical	**Exact**	**Numerical**	**Exact**
(0.1, 0.1)	0.48732	0.50010	0.49725	0.50000
(0.5, 0.1)	0.50002	0.50000	0.50025	0.50000
(0.9, 0.1)	0.50000	0.50000	0.49932	0.50000
(0.3, 0.3)	0.49531	0.50010	0.50687	0.50000
(0.7, 0.3)	0.50001	0.50000	0.49929	0.50000
(0.1, 0.5)	0.74990	0.75000	0.43945	0.50048
(0.5, 0.5)	0.49438	0.50010	0.49959	0.50000
(0.9, 0.5)	0.49977	0.50000	0.51376	0.50000
(0.3, 0.7)	0.75001	0.75000	0.41654	0.50048
(0.7, 0.7)	0.49324	0.50010	0.51050	0.50000
(0.1, 0.9)	0.75000	0.75000	0.75003	0.75000
(0.5, 0.9)	0.75001	0.75000	0.42895	0.50048
(0.9, 0.9)	0.47380	0.50010	0.56293	0.50000

Table 8
The numerical values for v in comparison with the exact solution at $t = 0.5$ and $t = 2$ with $\Delta t = 0.001$, and $\mathrm{Re} = 500$.

(x, y)	t=0.5		t=2	
	Numerical	**Exact**	**Numerical**	**Exact**
(0.1, 0.1)	1.01286	0.99990	1.00276	1.00000
(0.5, 0.1)	1.00000	1.00000	0.99976	1.00000
(0.9, 0.1)	1.00000	1.00000	1.00068	1.00000
(0.3, 0.3)	1.00481	0.99990	0.99313	1.00000
(0.7, 0.3)	0.99999	1.00000	1.00072	1.00000
(0.1, 0.5)	0.75010	0.75000	1.06055	0.99952
(0.5, 0.5)	1.00571	0.99990	1.00041	1.00000
(0.9, 0.5)	1.00022	1.00000	0.98624	1.00000
(0.3, 0.7)	0.74999	0.75000	1.08346	0.99952
(0.7, 0.7)	1.00676	0.99990	0.98950	1.00000
(0.1, 0.9)	0.75000	0.75000	0.74997	0.75000
(0.5, 0.9)	0.74999	0.75000	1.07105	0.99952
(0.9, 0.9)	1.02620	0.99990	0.93707	1.00000

Table 9
The numerical values for u in comparison with the exact solution at $t = 0.5$ and $t = 2$ with $\Delta t = 0.01$, and $\mathrm{Re} = 500$.

(x, y)	t=0.5		t=2	
	Numerical	**Exact**	**Numerical**	**Exact**
(0.1, 0.1)	0.48714	0.50010	0.49729	0.50000
(0.5, 0.1)	0.50002	0.50000	0.50024	0.50000
(0.9, 0.1)	0.50000	0.50000	0.49934	0.50000
(0.3, 0.3)	0.49519	0.50010	0.50690	0.50000
(0.7, 0.3)	0.50001	0.50000	0.49928	0.50000
(0.1, 0.5)	0.74990	0.75000	0.43939	0.50048
(0.5, 0.5)	0.49429	0.50010	0.49951	0.50000
(0.9, 0.5)	0.49978	0.50000	0.51355	0.50000
(0.3, 0.7)	0.75001	0.75000	0.41647	0.50048
(0.7, 0.7)	0.49325	0.50010	0.51008	0.50000
(0.1, 0.9)	0.75000	0.75000	0.75004	0.75000
(0.5, 0.9)	0.75001	0.75000	0.42909	0.50048
(0.9, 0.9)	0.47275	0.50010	0.56275	0.50000

Table 10
The numerical values for v in comparison with the exact solution at $t = 0.5$ and $t = 2$ with $\Delta t = 0.01$, and $\mathrm{Re} = 500$.

(x, y)	t=0.5		t=2	
	Numerical	**Exact**	**Numerical**	**Exact**
(0.1, 0.1)	1.01286	0.99990	1.00271	1.00000
(0.5, 0.1)	0.99999	1.00000	0.99976	1.00000
(0.9, 0.1)	1.00000	1.00000	1.00066	1.00000
(0.3, 0.3)	1.00481	0.99990	0.99310	1.00000
(0.7, 0.3)	0.99999	1.00000	1.00072	1.00000
(0.1, 0.5)	0.75010	0.75000	1.06061	0.99952
(0.5, 0.5)	1.00571	0.99990	1.00049	1.00000
(0.9, 0.5)	1.00022	1.00000	0.98646	1.00000
(0.3, 0.7)	0.74999	0.75000	1.08353	0.99952
(0.7, 0.7)	1.00676	0.99990	0.98992	1.00000
(0.1, 0.9)	0.75000	0.75000	0.74996	0.75000
(0.5, 0.9)	0.74999	0.75000	1.07091	0.99952
(0.9, 0.9)	1.02725	0.99990	0.93725	1.00000

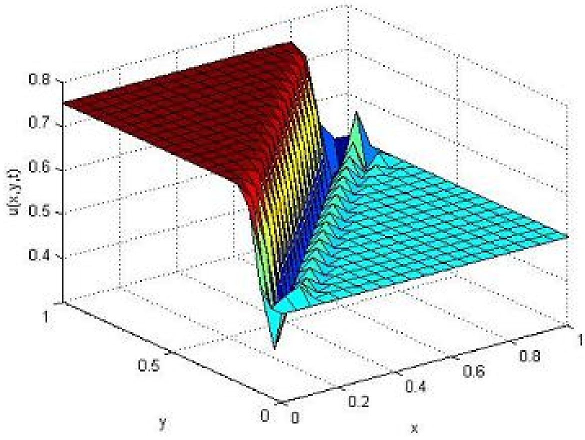

Fig.1.The numerical value of u for $\mathrm{Re} = 500$ at time level $t = 0.5$ with $\Delta t = 0.001$.

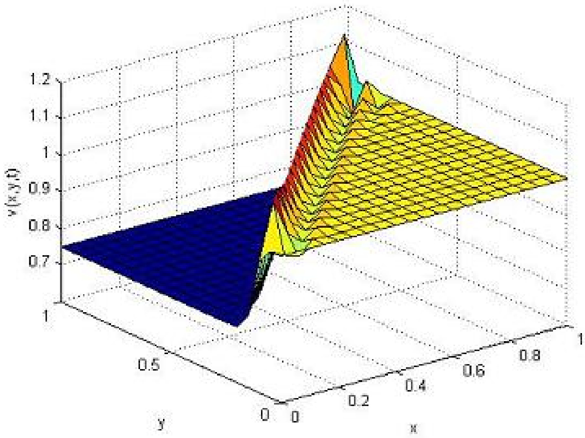

Fig.2.The numerical value of v for $\mathrm{Re} = 500$ at time level $t = 0.5$ with $\Delta t = 0.001$.

Table 11
Comparison of computed values of u for $\mathrm{Re} = 50$ at $t = 0.625$.

(x, y)	Computed values of u		
	Present work	A.R.Bahadir	Jain and Holla
	N=20	N=20	N=20
(0.1, 0.1)	0.97146	0.96688	0.97258
(0.3, 0.1)	1.15280	1.14827	1.16214
(0.2, 0.2)	0.86307	0.85911	0.86281
(0.4, 0.2)	0.97981	0.97637	0.96483
(0.1, 0.3)	0.66316	0.66019	0.66318
(0.3, 0.3)	0.77230	0.76932	0.77030
(0.2, 0.4)	0.58180	0.57966	0.58070
(0.4, 0.4)	0.75856	0.75678	0.74435

Table 12
Comparison of computed values of v for Re =50 at $t = 0.625$

(x, y)	Computed values of v		
	Present work	A.R.Bahadir	Jain and Holla
	N=20	N=20	N=20
(0.1, 0.1)	0.09869	0.09824	0.09773
(0.3, 0.1)	0.14158	0.14112	0.14039
(0.2, 0.2)	0.16754	0.16681	0.16660
(0.4, 0.2)	0.17110	0.17065	0.17397
(0.1, 0.3)	0.26378	0.26261	0.26294
(0.3, 0.3)	0.22654	0.22576	0.22463
(0.2, 0.4)	0.32851	0.32745	0.32402
(0.4, 0.4)	0.32500	0.32441	0.31822

Table 13
Comparison of computed values of u for Re =500 at $t = 0.625$.

(x, y)	Computed values of u		
	Present work	A.R.Bahadir	Jain and Holla
	N=20	N=20	N=20
(0.15, 0.1)	0.96870	0.96650	0.95691
(0.3, 0.1)	1.03200	1.02970	0.95616
(0.1, 0.2)	0.86178	0.84449	0.84257
(0.2, 0.2)	0.87814	0.87631	0.86399
(0.1, 0.3)	0.67920	0.67809	0.67667
(0.3, 0.3)	0.79947	0.79792	0.76876
(0.15, 0.4)	0.66036	0.54601	0.54408
(0.2, 0.4)	0.58959	0.58874	0.58778

Table 14
Comparison of computed values of v for $\mathrm{Re} = 500$ at $t = 0.625$.

(x, y)	Computed values of v		
	Present work	A.R.Bahadir	Jain and Holla
	N=20	N=20	N=20
(0.15, 0.1)	0.09043	0.09020	0.10177
(0.3, 0.1)	0.10728	0.10690	0.13287
(0.1, 0.2)	0.17295	0.17972	0.18503
(0.2, 0.2)	0.16816	0.16777	0.18169
(0.1, 0.3)	0.26268	0.26222	0.26560
(0.3, 0.3)	0.23550	0.23497	0.25142
(0.15, 0.4)	0.29019	0.31753	0.32084
(0.2, 0.4)	0.30419	0.30371	0.30927

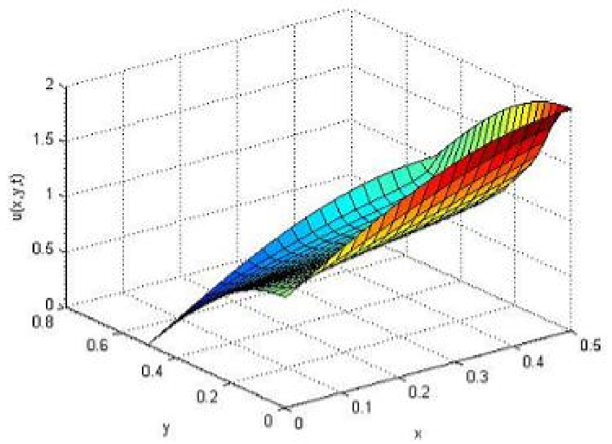

Fig.3.The computed value of u for $\mathrm{Re}=50$ at time level $t=0.625$.

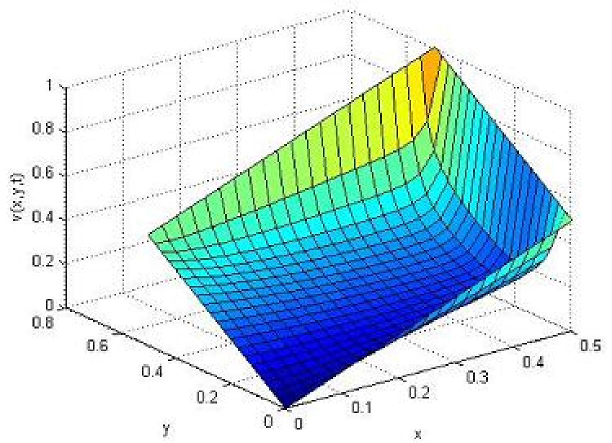

Fig.4.The computed value of v for $\mathrm{Re}=50$ at time level $t=0.625$.

4 Conclusion

Crank-Nicolson finite-difference method for two-dimensional coupled nonlinear viscous Burgers' equations has been presented.The advantage of the proposed method is that it is second order accurate in space and time. The efficiency and numerical accuracy of the present scheme are validated through two numerical examples. The test examples show that the present scheme is unconditionally stable as there is no constraint on time step-size. Numerical results are compared well with those from the exact solutions and previous available results.

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